Dáno:

$\overline{OA} = 500 \text{ mm}$

$\overline{AB} = 1500 \text{ mm}$

$\overline{CD} = 500 \text{ mm}$

$\overline{CL} = 500 \text{ mm}$

$\overline{DE} = 1000 \text{ mm}$

$\ell_5 = 1000 \text{ mm}$

$\varphi_{12} = \frac{1}{2}\alpha_{12}\cdot t^2 + \omega_{120}\cdot t + \varphi_{120}$

$\varphi_{120} = 135°$

$\omega_{120} = 1 \ \bar{s}^1$

$\alpha_{12} = 0{,}5 \ \bar{s}^2$
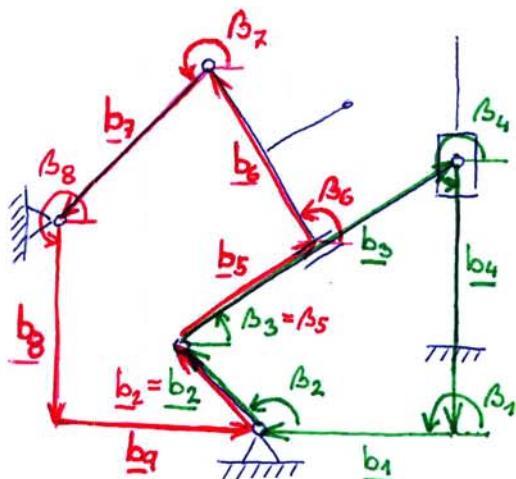
Určete:

polohu, rychlost, zrychlení členů mechanismu a bodu L

Řešení:

$n = 3(6-1) - 2(5+2+\varnothing) - 1\cdot\varnothing = 15 - 14 = 1°$ volnosti $(\rightarrow$ bude 1 nezávislá souřadnice$)$

$\ell = 7 + \varnothing - 6 + 1 = 2$ nezávislé smyčky $(\rightarrow 2$ vektorové rovnice $\rightarrow 4$ skalární rovnice $\rightarrow 4$ závislé souřadnice$)$

1. smyčka: $\underline{b_1} + \underline{b_2} + \underline{b_3} + \underline{b_4} = \varnothing$

2. smyčka: $\underline{b_2} + \underline{b_5} + \underline{b_6} + \underline{b_7} + \underline{b_8} + \underline{b_9} = \varnothing$

nezávislá s.: $\underline{q} = [\beta_2]$

závislé s.: $\underline{z} = \begin{bmatrix} \beta_3 \\ b_4 \\ b_5 \\ \beta_7 \end{bmatrix}$

závislosti:

$\beta_5 = \beta_3$

$\beta_6 = \beta_3 + \frac{\pi}{2}$

konstanty: $b_1, b_2, b_3, b_6, b_7, b_8, b_9, \beta_1, \beta_4, \beta_8, \beta_9$

skalární rovnice:

① x: $b_1 \cdot \cos\beta_1 + b_2 \cdot \cos\beta_2 + b_3 \cdot \cos\beta_3 + b_4 \cdot \cos\beta_4 = \varnothing$

y: $b_1 \cdot \sin\beta_1 + b_2 \cdot \sin\beta_2 + b_3 \cdot \sin\beta_3 + b_4 \cdot \sin\beta_4 = \varnothing$

② x: $b_2 \cdot \cos\beta_2 + b_5 \cdot \cos\beta_5 + b_6 \cdot \cos\beta_6 + b_7 \cdot \cos\beta_7 + b_8 \cdot \cos\beta_8 + b_9 \cdot \cos\beta_9 = \varnothing$

y: $b_2 \cdot \sin\beta_2 + b_5 \cdot \sin\beta_5 + b_6 \cdot \sin\beta_6 + b_7 \cdot \sin\beta_7 + b_8 \cdot \sin\beta_8 + b_9 \cdot \sin\beta_9 = \varnothing$

Rychlosti:

① $x:\ -b_2\sin\beta_2\cdot\dot\beta_2 - b_3\cdot\sin\beta_3\cdot\dot\beta_3 + \dot b_4\cdot\cos\beta_4 = \emptyset$

　$y:\ \ b_2\cos\beta_2\cdot\dot\beta_2 + b_3\cos\beta_3\cdot\dot\beta_3 + \dot b_4\sin\beta_4 = \emptyset$

② $x:\ -b_2\sin\beta_2\cdot\dot\beta_2 + \dot b_5\cos\beta_5 - b_5\sin\beta_5\cdot\overset{_{\nearrow\dot\beta_3}}{\dot\beta_5} - b_6\sin\beta_6\cdot\overset{_{\nearrow\dot\beta_3}}{\dot\beta_6} - b_7\sin\beta_7\cdot\dot\beta_7 = \emptyset$

　$y:\ \ b_2\cos\beta_2\cdot\dot\beta_2 + \dot b_5\sin\beta_5 + b_5\cos\beta_5\cdot\underset{_{\dot\beta_3}}{\dot\beta_5} + b_6\cos\beta_6\cdot\underset{_{\dot\beta_3}}{\dot\beta_6} + b_7\cos\beta_7\cdot\dot\beta_7 = \emptyset$

$$\underbrace{\begin{bmatrix} -b_3\cdot\sin\beta_3 & \cos\beta_4 & \emptyset & \emptyset \\ b_3\cdot\cos\beta_3 & \sin\beta_4 & \emptyset & \emptyset \\ -b_5\sin\beta_5 - b_6\cdot\sin\beta_6 & \emptyset & \cos\beta_5 & -b_7\sin\beta_7 \\ b_5\cos\beta_5 + b_6\cos\beta_6 & \emptyset & \sin\beta_5 & b_7\cos\beta_7 \end{bmatrix}}_{\underline{J_z}} \underbrace{\begin{bmatrix} \dot\beta_3 \\ \dot b_4 \\ \dot b_5 \\ \dot\beta_7 \end{bmatrix}}_{\underline{\dot z}} + \underbrace{\begin{bmatrix} -b_2\cdot\sin\beta_2 \\ b_2\cos\beta_2 \\ -b_2\sin\beta_2 \\ b_2\cos\beta_2 \end{bmatrix}}_{\underline{J_q}} \underbrace{\begin{bmatrix} \dot\beta_2 \end{bmatrix}}_{\underline{\dot q}} = \begin{bmatrix} \emptyset \\ \emptyset \\ \emptyset \\ \emptyset \end{bmatrix}$$

$$\underline{J_z}\cdot\underline{\dot z} + \underline{J_q}\cdot\underline{\dot q} = \underline\emptyset \ \Rightarrow\ \underline{\dot z} = -\underline{J_z}^{-1}\underline{J_q}\,\underline{\dot q}$$

Zrychlení: $\quad(\sim\!\!\sim\!\!\sim J_z,\ \text{mm}\ J_q,\ \underline{\quad} jqz)$

① $x:\ -b_2\cos\beta_2\cdot\dot\beta_2^2 - b_2\sin\beta_2\cdot\ddot\beta_2 - b_3\cos\beta_3\cdot\dot\beta_3^2 - b_3\sin\beta_3\cdot\ddot\beta_3 + \ddot b_4\cos\beta_4 = 0$

　$y:\ -b_2\sin\beta_2\cdot\dot\beta_2^2 + b_2\cos\beta_2\cdot\ddot\beta_2 - b_3\sin\beta_3\cdot\dot\beta_3^2 + b_3\cos\beta_3\cdot\ddot\beta_3 + \ddot b_4\sin\beta_4 = \emptyset$

② $x:\ -b_2\cos\beta_2\cdot\dot\beta_2^2 - b_2\sin\beta_2\cdot\ddot\beta_2 + \ddot b_5\cos\beta_5 - \dot b_5\sin\beta_5\cdot\dot\beta_5 - \dot b_5\sin\beta_5\cdot\dot\beta_5 - b_5\cos\beta_5\cdot\dot\beta_5^2 -$
　$\quad - b_5\sin\beta_5\cdot\ddot\beta_5 - b_6\cos\beta_6\cdot\dot\beta_6^2 - b_6\sin\beta_6\cdot\ddot\beta_6 - b_7\cos\beta_7\cdot\dot\beta_7^2 - b_7\sin\beta_7\cdot\ddot\beta_7 = \emptyset$
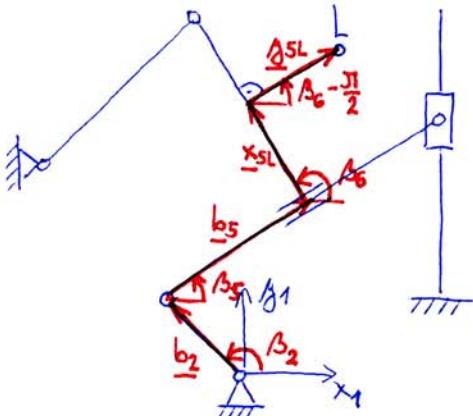
　$y:\ -b_2\sin\beta_2\cdot\dot\beta_2^2 + b_2\cos\beta_2\cdot\ddot\beta_2 + \ddot b_5\sin\beta_5 + \dot b_5\cos\beta_5\cdot\dot\beta_5 + \dot b_5\cos\beta_5\cdot\dot\beta_5 - b_5\sin\beta_5\cdot\dot\beta_5^2 +$
　$\quad + b_5\cos\beta_5\cdot\ddot\beta_5 - b_6\sin\beta_6\cdot\dot\beta_6^2 + b_6\cos\beta_6\cdot\ddot\beta_6 - b_7\sin\beta_7\cdot\dot\beta_7^2 + b_7\cos\beta_7\cdot\ddot\beta_7 = \emptyset$

$$jqz = \begin{bmatrix} -b_2\cos\beta_2\cdot\dot\beta_2^2 - b_3\cos\beta_3\cdot\dot\beta_3^2 \\ -b_2\sin\beta_2\cdot\dot\beta_2^2 - b_3\sin\beta_3\cdot\dot\beta_3^2 \\ -b_2\cos\beta_2\cdot\dot\beta_2^2 - 2\dot b_5\sin\beta_5\cdot\dot\beta_5 - b_5\cos\beta_5\cdot\dot\beta_5^2 - b_6\cos\beta_6\cdot\dot\beta_6^2 - b_7\cos\beta_7\cdot\dot\beta_7^2 \\ -b_2\sin\beta_2\cdot\dot\beta_2^2 + 2\dot b_5\cos\beta_5\cdot\dot\beta_5 - b_5\sin\beta_5\cdot\dot\beta_5^2 - b_6\sin\beta_6\cdot\dot\beta_6^2 - b_7\sin\beta_7\cdot\dot\beta_7^2 \end{bmatrix}$$
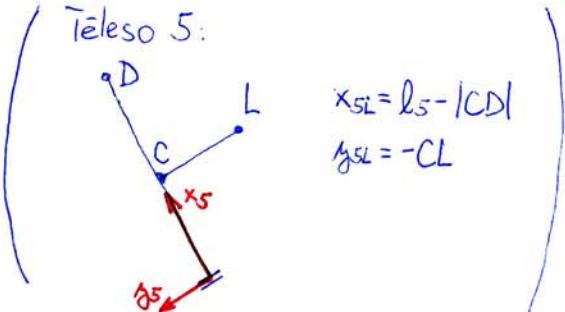
$$(\dot\beta_5 = \dot\beta_3,\ \ddot\beta_5 = \ddot\beta_3,\ \dot\beta_6 = \dot\beta_3,\ \ddot\beta_6 = \ddot\beta_3)$$

$$\underline{J_z}\cdot\underline{\ddot z} + \underline{J_q}\cdot\underline{\ddot q} + \underline{jqz} = \underline\emptyset \ \Rightarrow\ \underline{\ddot z} = -\underline{J_z}^{-1}\left(\underline{J_q}\,\underline{\ddot q} + \underline{jqz}\right)$$

Bod L:

$$\underline{r}_{1L} = \underline{b}_2 + \underline{b}_5 + \underline{x}_{5L} + \underline{y}_{5L}$$

Těleso 5:

$$x_{5L} = \ell_5 - |CD|$$
$$y_{5L} = -CL$$

Poloha:

$$x_{1L} = b_2 \cos\beta_2 + b_5 \cdot \cos\beta_5 + x_{5L}\cos\beta_6 + |y_{5L}|\cos\left(\beta_6 - \frac{\pi}{2}\right) \quad (*)$$

$$y_{1L} = b_2 \sin\beta_2 + b_5 \sin\beta_5 + x_{5L}\sin\beta_6 + |y_{5L}|\sin\left(\beta_6 - \frac{\pi}{2}\right)$$

Rychlost:

$$v_{1Lx} = \dot{x}_{1L} = -b_2 \cdot \sin\beta_2 \cdot \dot{\beta}_2 + \dot{b}_5 \cos\beta_5 - b_5 \sin\beta_5 \dot{\beta}_5 - x_{5L}\sin\beta_6 \cdot \dot{\beta}_6 - |y_{5L}|\sin\left(\beta_6 - \frac{\pi}{2}\right)\cdot\dot{\beta}_6$$

$$v_{1Ly} = \dot{y}_{1L} = b_2 \cdot \cos\beta_2 \cdot \dot{\beta}_2 + \dot{b}_5 \sin\beta_5 + b_5 \cos\beta_5 \cdot \dot{\beta}_5 + x_{5L}\cos\beta_6 \cdot \dot{\beta}_6 + |y_{5L}|\cdot\cos\left(\beta_6 - \frac{\pi}{2}\right)\cdot\dot{\beta}_6$$

Zrychlení:

$$a_{1Lx} = \ddot{x}_{1L} = -b_2 \cdot \cos\beta_2 \cdot \dot{\beta}_2^2 - b_2 \cdot \sin\beta_2 \cdot \ddot{\beta}_2 + \ddot{b}_5 \cos\beta_5 - \dot{b}_5 \sin\beta_5 \cdot \dot{\beta}_5 - \dot{b}_5 \sin\beta_5 \cdot \dot{\beta}_5 - b_5 \cdot \cos\beta_5 \cdot \dot{\beta}_5^2 -$$
$$- b_5 \cdot \sin\beta_5 \cdot \ddot{\beta}_5 - x_{5L}\cdot\cos\beta_6 \cdot \dot{\beta}_6^2 - x_{5L}\cdot\sin\beta_6 \cdot \ddot{\beta}_6 - |y_{5L}|\cdot\cos\left(\beta_6 - \frac{\pi}{2}\right)\cdot\dot{\beta}_6^2 - |y_{5L}|\cdot\sin\left(\beta_6 - \frac{\pi}{2}\right)\cdot\ddot{\beta}_6$$

$$a_{1Ly} = \ddot{y}_{1L} = -b_2 \cdot\sin\beta_2 \cdot \dot{\beta}_2^2 + b_2 \cdot\cos\beta_2 \cdot \ddot{\beta}_2 + \ddot{b}_5 \sin\beta_5 + \dot{b}_5 \cdot\cos\beta_5 \cdot \dot{\beta}_5 + \dot{b}_5 \cos\beta_5 \cdot \dot{\beta}_5 - b_5 \cdot\sin\beta_5 \cdot \dot{\beta}_5^2 +$$
$$+ b_5 \cdot\cos\beta_5 \cdot \ddot{\beta}_5 - x_{5L}\sin\beta_6 \cdot \dot{\beta}_6^2 + x_{5L}\cdot\cos\beta_6 \cdot \ddot{\beta}_6 - |y_{5L}|\cdot\sin\left(\beta_6 - \frac{\pi}{2}\right)\cdot\dot{\beta}_6^2 + |y_{5L}|\cos\left(\beta_6 - \frac{\pi}{2}\right)\cdot\ddot{\beta}_6$$

(*) Pozn.

Často se používá i zápis ve tvaru:

$$x_{1L} = b_2 \cdot\cos\beta_2 + b_5 \cdot\cos\beta_5 + x_{5L}\cos\beta_6 - y_{5L}\cdot\sin\beta_6$$

$$y_{1L} = b_2 \cdot\sin\beta_2 + b_5 \cdot\sin\beta_5 + x_{5L}\sin\beta_6 + y_{5L}\cos\beta_6$$