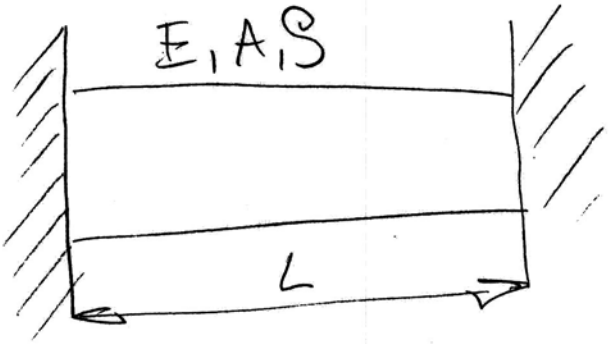
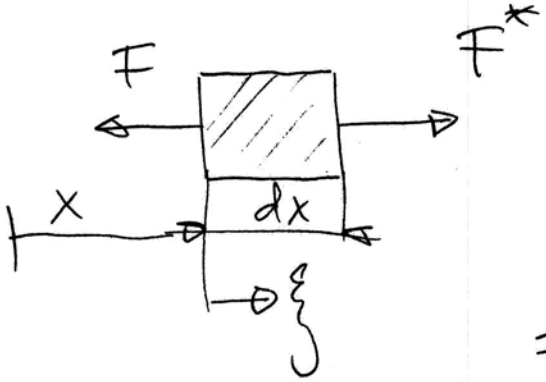


D: E, A, S, ℓ

U: diferenc. rovnice

• okr. tvary kmitání



$$\underbrace{m \cdot a}_{} = F$$

$$A \cdot S \cdot dx \cdot \frac{\partial^2 \xi}{\partial t^2} = -F + F^*$$

$$F^* = F + \frac{\partial F}{\partial x} dx$$

$$\sigma = E \cdot \varepsilon = \frac{F}{A} \Rightarrow F = E \cdot A \cdot \varepsilon = E \cdot A \cdot \frac{\partial \xi}{\partial x}$$

$$F = E \cdot A \frac{\partial \xi}{\partial x}$$

$$\frac{\partial F}{\partial x} = E \cdot A \frac{\partial^2 \xi}{\partial x^2}$$

$$\cancel{A} S \cdot \cancel{dx} \frac{\partial^2 \xi}{\partial t^2} = \cancel{F} + \cancel{F} + E \cancel{A} \frac{\partial^2 \xi}{\partial x^2} \cancel{dx}$$

$$\boxed{\frac{\partial^2 \xi}{\partial t^2} = \frac{E}{\rho} \frac{\partial^2 \xi}{\partial x^2}}$$

vlnová rovnice pro $\xi = \xi(x, t)$

$y = \sin \Omega t$

$\ddot{y} = -\Omega^2 \underbrace{\sin \Omega t}_{y}$

$\frac{\ddot{y}}{y} = -\Omega^2$

Metoda separace fcí $\xi = X(x) T(t)$

$$\frac{\partial^2 \xi}{\partial t^2} = X(x) \cdot \ddot{T}(t)$$

$$\frac{\partial^2 \xi}{\partial x^2} = X''(x) \cdot T(t)$$

$$X \ddot{T} = \underbrace{\frac{E}{\rho}}_{-C_0^2} X'' T \quad \Big/ \frac{1}{XT}$$

$$\underbrace{\frac{\ddot{T}}{T} = \underbrace{-C_0^2 \frac{X''}{X}}_{\text{2. rovnice}}}_{\text{1. rovnice}} = -\Omega_0^2$$

2 obyč. dif. rce

$$\ddot{T} + \Omega_0^2 T = 0 \rightarrow T(t) = A\cos\Omega_0 t + B\sin\Omega_0 t$$

$$X'' + \underbrace{\frac{\Omega_0^2}{c_0^2}}_{\alpha^2} X = 0 \rightarrow X(x) = C\cos\alpha x + D\sin\alpha x$$

A, B → z poč. podmínek

C, D → z okrajových podmínek

$$\left.\begin{aligned} \xi(0,t) &= X(0)T(t) = 0 \\ \xi(L,t) &= X(L)T(t) = 0 \end{aligned}\right\} \quad \begin{aligned} 0 &= C\cos\alpha\cdot 0 + D\sin\alpha\cdot 0 \\ 0 &= C\cos\alpha\cdot L + D\sin\alpha\cdot L \end{aligned}$$

$$\begin{bmatrix} \overset{\uparrow 1}{\cos 0} & \overset{\uparrow 0}{\sin 0} \\ \cos\alpha L & \sin\alpha L \end{bmatrix} \begin{bmatrix} C \\ D \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \rightarrow 1\cdot C + 0\cdot D = 0 \Rightarrow C = 0$$

řešení $\det[\ ] = 0$

$\Downarrow$

$$1\cdot\sin\alpha\cdot L - 0 = 0 \Rightarrow \sin\alpha\cdot L = 0 \text{ pro } \alpha L = n\cdot\pi$$

$$n = 1, 2, \dots, \infty$$

$$\Rightarrow \alpha = \frac{n\cdot\pi}{L}$$

$$\sqrt{\frac{E}{\rho}}$$

$$\boxed{\Omega_n = c_0 \frac{\pi}{L}\cdot n} \Rightarrow \frac{\Omega_n}{c_0} = \alpha = \frac{n\pi}{L}$$

Vlastní tvary kmitu

$$\Phi_n(x) = C_n \cos \alpha x + D_n \sin \alpha x$$

$$\alpha = \frac{n \cdot \pi}{L} \qquad C_n \to \emptyset$$

$$\Phi_n(x) = C_n \cos\left(\frac{n \cdot \pi}{L} \cdot x\right) + D_n \sin\left(\frac{n\pi}{L} \cdot x\right)$$

$$C_n \to \emptyset$$

$$\Phi_n(x) = D_n \sin\left(\frac{n\pi}{L} \cdot x\right)$$