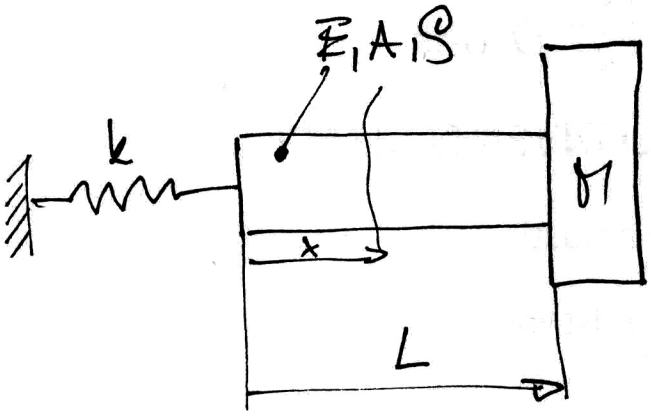
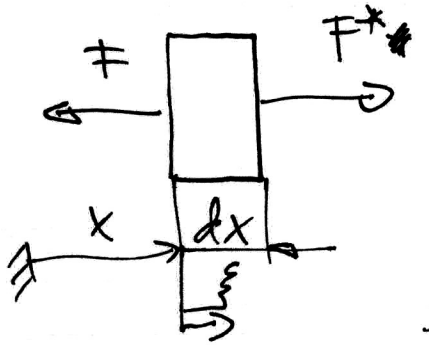


D: A, E, S, L, k, M

U:

1) sestavte frekvenční rovnici pro podélné kmity 1D kontinua
2) vyjádřete deformaci



$$m \cdot a = \sum F$$

$$\rho A\,dx \cdot \frac{\partial^2 \xi}{\partial t^2} = F^* - F$$

$$\rho A\,dx \frac{\partial^2 \xi}{\partial t^2} = F(x) + \frac{\partial F}{\partial x}dx - F(x)$$

$$\sigma = E \cdot \varepsilon \rightarrow \frac{F}{A} = E\frac{\partial \xi}{\partial x} \rightarrow F(x) = EA\frac{\partial \xi}{\partial x}$$

Pro jednorozměrnou napjatost

$$\rho A\,dx \frac{\partial^2 \xi}{\partial t^2} = \frac{\partial}{\partial x}\left(EA\frac{\partial \xi}{\partial x}\right)dx,$$

pro $E, A = \text{konst}$

$$\frac{\partial^2 \xi}{\partial t^2} = \frac{E}{\rho}\frac{\partial^2 \xi}{\partial x^2} \Rightarrow \frac{\partial^2 \xi}{\partial t^2} = c_0^2 \frac{\partial^2 \xi}{\partial x^2}\,; \quad c_0 = \sqrt{\frac{E}{\rho}}$$

vlnová rovnice

c_0 ... rychlost šíření vlny

Ustálený řešení

$$\xi(x,t) = X(x)\cdot T(t)$$

$$\Rightarrow X\ddot{T} = c_0^2 X'' T \;/\cdot \frac{1}{XT}$$

$$\frac{\ddot{T}}{T} = c_0^2 \frac{X''}{X} \Rightarrow \text{pokud } c_0^2 = \text{konst a obě}$$

strany se mají být rovny

$\Rightarrow$ jak, když obě jsou konstantní

v čase (t) i prostoru (x), např. $-\Omega_0^2$

$$\frac{\ddot{T}}{T} = c_0^2 \frac{X''}{X} = -\Omega_0^2$$

rozepíšeme do dvou rovnic

$$\frac{\ddot{T}}{T} = -\Omega_0^2 \Rightarrow \ddot{T} + \Omega_0^2 T = 0$$

$$c_0^2 \frac{X''}{X} = -\Omega_0^2 \Rightarrow X'' + \left(\frac{\Omega_0}{c_0}\right)^2 X = 0$$

α^2

$$T(t) = A_1 \cos \Omega_0 t + B_1 \sin \Omega_0 t$$

$$X(x) = A_2 \cos \alpha x + B_2 \sin \alpha x$$

$$\xi(x,t) = (A_1 \cos \Omega_0 t + B_1 \sin \Omega_0 t)(A_2 \cos \alpha x + B_2 \sin \alpha x)$$

$$X'(x) = -A_2 \sin(\alpha x)\cdot \alpha + B_2 \cdot \alpha \cos(\alpha x)$$

Одређење решења

← одрон решења

$x=0 \quad EA\frac{\partial \xi}{\partial x} = k\cdot\xi$ (силама)

$x=L \quad EA\frac{\partial \xi}{\partial x} = -M\ddot{\xi}$ (силама) ← Д'Аламбер. инерције (дељ.)

$\left(\sigma = E\cdot\varepsilon \Rightarrow \frac{F}{A} = E\frac{\partial \xi}{\partial x}\right)$

$\left(F(x) = EA\frac{\partial \xi}{\partial x}\right)$

$$\left.\begin{aligned} EA\,X(0)'T &= k\,X(0)T \\ EA\,X'(L)T &= -M\,X(L)\ddot{T} \end{aligned}\right\} \quad \begin{aligned} EA\,X'(0) &= k\,X(0) \\ EA\,X'(L) &= \Omega_0^2 M\,X(L) \end{aligned}$$

$$\ddot{T} + \Omega_0^2 T = 0 \Rightarrow \ddot{T} = -\Omega_0^2 T$$

Претпоставимо

$x=0$: $EA\left(-\alpha A_2 \underbrace{\sin(\alpha\cdot 0)}_{0} + B_2\alpha \underbrace{\cos(\alpha\cdot 0)}_{1}\right) = k\cdot[$

$[A_2 \underbrace{\cos\alpha\cdot 0}_{1} + B_2 \underbrace{\sin\alpha\cdot 0}_{0}]$

$x=L$: $EA\left[-\alpha A_2\sin(\alpha L) + B_2\alpha\cos(\alpha L)\right] = \Omega_0^2 M\left[A_2\cos(\alpha L) + B_2\sin(\alpha L)\right]$

$$\begin{bmatrix} k & -EA\alpha \\ \Omega_0^2 M\cos(\alpha L) + EA\alpha\sin(\alpha L) & \Omega_0^2 M\sin(\alpha L) - EA\alpha\cos(\alpha L) \end{bmatrix}\begin{bmatrix} A_2 \\ B_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

→ фреквентна једначина

$$k\left(\Omega_0^2 M\sin(\alpha L) - EA\alpha\cos(\alpha L)\right) + EA\alpha\left(\Omega_0^2 M\cos(\alpha L) + EA\alpha\sin(\alpha L)\right) = 0$$

$f(\alpha_n) = 0$, $\alpha_n = \frac{\Omega_n}{c_0}$

Карактеристични

$\alpha_{1,2,\ldots,\infty}$, $\Omega_{1,2,\ldots,\infty}$

Nalezení α_n, $n=1,2,\dots,\infty$ (lin. záv. rovnice)

$$k A_{2n} - EA\alpha_n B_{2n} = 0$$

volíme $B_{2n}=1 \Rightarrow A_{2n} = \frac{EA\alpha_n}{k}$

Výsledné řešení:

$$\xi(x,t) = \sum_{n=1}^{\infty} \left[\cos(\alpha_n \cdot x) \frac{EA\alpha_n}{k} + \sin(\alpha_n x) \right] (A_n \cos\Omega_n t + B_n \sin\Omega_n t)$$

$\Rightarrow$ konstanty A_n, B_n z počátečních podmínek